

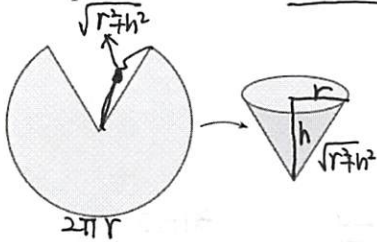
Week 8 Thursday - some hard problems in HW

**Instructions:** These hw problems are by no means easy. The plan is to go over at least one in discussion. For the rest, the solutions will be posted after class.

Don't understand why this problem make the calculation so complicated ... — P: !!

1. (Written HW 1029 4.6 #22)

A cone-shaped drinking cup is to be made to hold  $27 \text{ cm}^3$  of water. Find the height and radius of the cup that will use the smallest amount of paper.



Constraint

$$\frac{1}{3} \pi r^2 h = 27$$

$$\Rightarrow h = \frac{81}{\pi r^2}$$

Formula

$$A = \frac{1}{2} \sqrt{r^2+h^2} \cdot 2\pi r$$

$$= \pi r \sqrt{r^2+h^2}$$

$$= \pi r (r^2+h^2)^{\frac{1}{2}}$$

$$A = \pi r \left( r^2 + \frac{81^2}{\pi^2 r^4} \right)^{\frac{1}{2}}$$

$$A' = \pi \left( r^2 + \frac{81^2}{\pi^2 r^4} \right)^{\frac{1}{2}} + \pi r \cdot \frac{1}{2} \left( r^2 + \frac{81^2}{\pi^2 r^4} \right)^{-\frac{1}{2}} \cdot \left( 2r - \frac{324}{\pi^2 r^5} \right)$$

$$= \pi \sqrt{r^2 + \frac{81^2}{\pi^2 r^4}} + \frac{\pi r}{2} \frac{2r - \frac{324}{\pi^2 r^5}}{\sqrt{r^2 + \frac{81^2}{\pi^2 r^4}}} = 0$$

$$r^2 \frac{81^2}{\pi^2 r^4} + r^2 - \frac{2 \times 81^2}{\pi^2} \frac{1}{r^4} = 0$$

$$2r^2 - \frac{81^2}{\pi^2 r^4} = 0$$

$$r^6 = \frac{81^2}{2\pi^2} \Rightarrow r = \sqrt[6]{\frac{81^2}{2\pi^2}}$$

$$h = \frac{81}{\pi \sqrt[3]{\frac{81^2}{2\pi^2}}}$$

2. (Written HW 1029 4.7 #1)

During the summer months Terry makes and sells necklaces on the beach. Last summer he sold the necklaces for \$10 each and his sales averaged 20 per day. When he increased the price by \$1, he found that he lost two sales per day. (a) Find the demand function, assuming that it is linear. (b) If the material for the necklace costs Terry \$6, what should the selling price be to maximize his profit?

\$10 → #20 necklaces.  
 +\$1 → - #2

For a similar problem, check textbook *BSS Example 3* (with solutions).

q: necklace.

(a)

$$D(q) = \frac{20-q}{2} + 10$$

(b)  $P(q) = q \left( \frac{20-q}{2} + 10 \right) - 6q = q \left( 20 - \frac{1}{2}q \right) - 6q = 14q - \frac{1}{2}q^2$

$$P'(q) = 0 \Rightarrow 14 - q = 0$$

$$q = 14$$

$$D(14) = \frac{20-14}{2} + 10 = 13 \text{ selling price.}$$

3. (Webwork 1026 #3.)

Find the maximum area of a triangle formed in the first quadrant by the  $x$ -axis,  $y$ -axis and a tangent line to the graph of  $f = (x+10)^{-2}$ .

pt:  $(a, \frac{1}{(a+10)^2})$

$$f'(x) = -2(x+10)^{-3} = -\frac{2}{(x+10)^3}$$

$$f'(a) = -\frac{2}{(a+10)^3}$$

Tangent line:  $y - \frac{1}{(a+10)^2} = -\frac{2}{(a+10)^3}(x-a)$

$$x=0 \Rightarrow y = \frac{1}{(a+10)^2} - \frac{2}{(a+10)^3}(-a) = \frac{a+10}{(a+10)^3} + \frac{2a}{(a+10)^3} = \frac{3a+10}{(a+10)^3}$$

$$y=0 \Rightarrow x = \frac{3a+10}{2}$$

(because  $\frac{1}{(a+10)^2} = \frac{2}{(a+10)^3}(x-a) \Rightarrow a+10 = 2(x-a) \Rightarrow a+10 = 2x-2a$ )

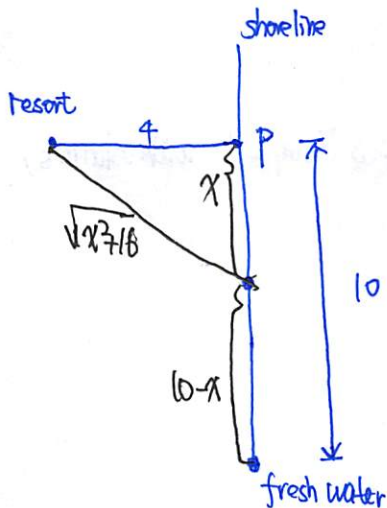
$$A = \frac{1}{2}xy = \frac{1}{2} \cdot \frac{3a+10}{2} \cdot \frac{3a+10}{(a+10)^3} = \frac{1}{4} \frac{(3a+10)^2}{(a+10)^3}$$

$$A' = \frac{1}{4} \frac{2(3a+10) \cdot 3(a+10)^3 - 3(a+10)^2(3a+10)^2}{(a+10)^6} = 0$$

$$\Rightarrow 2(3a+10) \cdot 3(a+10)^3 = 3(a+10)^2(3a+10)^2 \Rightarrow 2a+20 = 3a+10$$

4. (Webwork 1026 #4.)

A small resort is situated on an island that lies exactly 4 miles from P, the nearest point to the island along a perfectly straight shoreline. 10 miles down the shoreline from P is the closest source of fresh water. If it costs 1.7 times as much money to lay pipe in the water as it does on land, how far down the shoreline from P should the pipe from the island reach land in order to minimize the total construction costs?



Formula

$$C = 1.7\sqrt{x^2+16} + 10-x \quad (0 \leq x \leq 10)$$

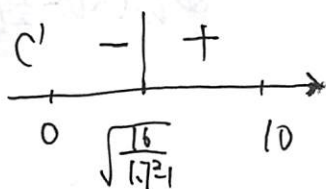
$$C' = 1.7 \cdot \frac{1}{2} (x^2+16)^{-\frac{1}{2}} \cdot 2x - 1 = 0$$

$$1.7 \frac{x}{\sqrt{x^2+16}} = 1$$

$$1.7x = \sqrt{x^2+16}$$

$$1.7^2 x^2 = x^2+16$$

$$x = \sqrt{\frac{16}{1.7^2-1}}$$



Down the shoreline from P  $\sqrt{\frac{16}{1.7^2-1}}$  miles, total cost is minimized.

$$\Rightarrow A = \frac{1}{4} \cdot \frac{40^2}{20^3} = \frac{1}{20}$$

